NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1081

FLOW OVER A SLENDER BODY OF REVOLUTION AT SUPERSONIC VELOCITIES

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Washington
August 1946

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SUMMARY

The theory of small disturbances is applied to the calculation of the pressure distribution and drag of a closed body of revolution traveling at supersonic speeds. It is shown that toward the rear of the body the shape of the pressure distribution is similar to that for subsonic flow. For fineness ratios between 10 and 15 the theoretical wave drag is of the same order as probable values of the frictional drag.

INTRODUCTION

Methods for calculation of the flow over a body of revolution traveling at supersonic velocities have been known for some time. (See references 1 and 2.) Investigations along these lines have, however, been confined chiefly to bodies having the form of artillery projectiles. Such bodies, because of their blunt forms, show relatively high drags and are thus not suited for use on high-speed aircraft. The drag of slender bodies and the effects of fairing the rear of these bodies are therefore of considerable interest in connection with the problem of flight at speeds above the speed of sound.

In view of the interest in possible aeronautical applications it was thought worth while to apply the known methods to a particular case of a closed body having both a tapered nose and a tapered tail. Slender shapes described by the rotation of parabolic arcs were chosen and the resulting pressure distributions over the surfaces as well as along the axes behind the bodies were calculated. The results are compared with those obtained for similar shapes in an incompressible fluid and also in one case with a two-dimensional body having a similar cross section.

SYMBOLS

x, y, z	Cartesian coordinates
V	undisturbed fluid velocity
a	speed of sound in fluid
M	Mach number (V/a)
$B = \sqrt{M^2 - 1}$	
$r = \sqrt{y^2 + z^2}$	
ρ	density of fluid
q	dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$
Δp	pressure increment
ϕ_o	velocity potential of single source
ϕ	velocity potential of continuous distribution of sources along x-axis
ξ	abscissa of individual sources
R	radius of body
d	maximum diameter of body
L	length of body
S_{max}	maximum cross-sectional area
D	drag
C_{D_a}	drag coefficient based on maximum cross-sectional area (D/qS_{max})
$C_{D_{vol}}$	drag coefficient based on 2/3 power of volume $\left(\frac{D}{q(\text{Volume})^{2/3}}\right)$
c, C	constants

METHODS OF CALCULATION

The method used herein follows closely that of reference 1. Figure 1 shows the shape of the body and the orientation of the axes. The disturbance produced by the body is assumed to be small and the flow isentropic so that the linearized equation for the potential of the disturbance velocities ϕ will apply. This equation is (see reference 2)

$$\left(1 - M^2\right)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{1}$$

As in the case of an incompressible fluid the flow over the body can be obtained by the addition of flows due to an infinite number of sources distributed along the axis. The potential of a single source in a supersonic stream is

$$\phi_o = \frac{CV}{\sqrt{x^2 - B^2(y^2 + z^2)}} = \frac{CV}{\sqrt{x^2 - B^2r^2}} \tag{2}$$

where

$$B = \sqrt{M^2 - 1}$$

and

$$r = \sqrt{y^2 + z^2}$$

Figure 2 shows the equipotential lines for the supersonic source compared with those for a source in an incompressible flow. In the case of a source in an incompressible flow the equipotential surfaces are spheres, given by the expression

$$\phi_o = \frac{CV}{\sqrt{x^2 + y^2 + z^2}} \tag{3}$$

In the supersonic case the equipotential surfaces are hyperboloids of two sheets contained within the Mach cones. Although the mathematical expression has values in two cones, one ahead of and one behind the source, only the values behind have physical significance.

It will be noted that the distribution of velocities along the x-axis is the same for the supersonic source (equation (2)) as for the subsonic or incompressible source (equation (3)). Since the forward cone is to be disregarded in the supersonic case, however, it is found that the coefficient C in equation (2) must be doubled in order to produce the same flux, or intensity, as equation (3). The result is that the velocities along the axis behind a supersonic source are exactly twice those of a subsonic source having the same intensity.

The sources and sinks are assumed to be continuously distributed with intensity $2\pi Vf(\xi)$ per unit length along the x-axis from -1 to 1. The abscissas of the individual sources are denoted by ξ. Positive values of $f(\xi)$ denote sources and negative values denote sinks. By adding the potentials due to the single elementary sources $f(\xi)d\xi$ the resulant flow

$$\phi = V\int_{-1}^{x-Br} \frac{f(\xi)d\xi}{\sqrt{(x-\xi)^2 - B^2r^2}} \tag{4}$$

is obtained.

The problem is to determine a source distribution in such a way that

$$\frac{1}{V}\left(\frac{\partial\phi}{\partial r}\right)_{r=R} = \frac{dR}{dx} \tag{5}$$

where dR/dx gives the shape of the meridian curve of the body of revolution. It is shown in reference 1 that to a first approximation for a slender body the source strength is proportional to the rate of change of the cross section of the body, that is

$$f(x) = R\frac{dR}{dx} \tag{6}$$

A similar approximation can be applied to obtain the source distribution for a body in subsonic flow. The distributions are, in fact, the same in the two cases with the exception that in the supersonic flow the value of $f(x)$ must be doubled to account for the elimination of the flux through the forward cones.

By choosing $f(x) = c(x^3 - x)$ and solving equation (6), the following expression for R was obtained:

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$$R = \sqrt{\frac{c}{2}}\,(1 - x^2) \tag{7}$$

This expression may be recognized as the equation of a surface obtained by revolving a parabolic arc about its chord. The fineness ratio of the body is determined by the value assigned the factor c.

On substituting $c(\xi^3 - \xi)$ for $f(\xi)$ in equation (4), the velocity increment $\partial\phi/\partial x$ at point (x,r) is found to be (see equation (9.5), p.39, reference 1)

$$\frac{\partial\phi}{\partial x} = V\int_{-1}^{x-Br} \frac{f'(\xi)\,d\xi}{\sqrt{(x-\xi)^2 - B^2r^2}}$$

$$= cV\int_{-1}^{x-Br} \frac{(3\xi^2 - 1)}{\sqrt{(x-\xi)^2 - B^2r^2}}\,d\xi$$

$$= cV\left[\frac{(3 - 9x)}{2}\sqrt{(x+1)^2 - B^2r^2} + \left(3x^2 - 1 + \frac{3}{2}B^2r^2\right)\cosh^{-1}\left(\frac{x+1}{Br}\right)\right] \tag{8}$$

over the body.

Along the axis behind the body the integration gives

$$cV\left[(3x^2 - 1)\log\frac{x+1}{x-1} - 6x\right] \tag{9}$$

The pressure coefficients were calculated by the formula

$$\frac{\Delta p}{q} = \frac{2}{V}\frac{d\phi}{dx} \tag{10}$$

RESULTS

Calculations have been made for a Mach number of 1.4 and for three thickness ratios d/L of 0.0667, 0.10, and 0.15 corresponding to fineness ratios L/d of 15, 10, and 6.67, respectively. The results for the three bodies are shown in figures 3, 4, and 5, respectively, and are compared with the theoretical pressure distributions over these bodies in an incompressible fluid. A discussion of the errors involved in the linear theory and the variation of the pressures with Mach number will be found in reference 3.

Comparison of the distributions in a compressible fluid and in an incompressible fluid shows a certain similarity, especially toward the rear of the body. The effect of supersonic speed appears to be similar to the effect of a lag inasmuch as the negative pressure peak and the region of pressure recovery are displaced rearward. The pressures along the axis behind the body are just twice those produced by an incompressible fluid, as may be seen by referring to the velocity field of a single source.

The results obtained herein for the three-dimensional body are in marked contrast to the results obtained for two-dimensional bodies, or wing sections, having similar cross sections. As is well known, in the two-dimensional case no pressure recovery takes place at supersonic speeds, the pressure at a point being determined solely by the inclination of the surface at that point so that positive pressures occur wherever the cross section is expanding and negative pressures occur wherever the cross section is diminishing. Figure 6 shows the comparison of the two-dimensional and three-dimensional bodies for the 0.10 thickness ratio.

The essential difference between the two- and three-dimensional flows corresponds to the difference noted by Lamb (reference 4) between the characteristics of a plane sound wave and an axially symmetrical wave diverging from a center. As noted by Lamb, the plane wave, which corresponds in the present case to the flow produced by the two-dimensional wing section, is propagated indefinitely without change of form; whereas the axially symmetrical wave, which approximates that produced by an element of the slender body of revolution, does not follow the form of the disturbing motion but leaves a "tail" of diminishing intensity and indefinite extent. Thus the wing section leaves no pressure disturbance in its wake, whereas the axially symmetrical body is followed by an indefinite region of positive pressure. Integration of the axial components of the pressures acting on the body, however, shows that the positive pressure at the rear of the three-dimensional body is sufficient to cancel only a small fraction of the total pressure or wave drag.

The wave-drag coefficients based on the maximum frontal area were found to be 0.049, 0.11, and 0.24 for the bodies with thickness ratios of 0.0667, 0.10, and 0.15, respectively. Figure 7 shows a comparison of these values with the wave drags of corresponding two-dimensional wing sections. It will be noted that the wave drag of the fuselage form is approximately proportional to the square of the thickness ratio.

An approximate estimate of the total drag of a body may be obtained by adding values of the frictional drag to the wave drag. A rather complete treatment of the frictional drag of bodies of revolution at subsonic speeds is available from reference 5. By use of values from reference 5 corresponding to a fully turbulent boundary layer and a Reynolds number of 10^8, the following estimates of the total drags of the bodies were obtained:

Thickness ratio	C_{D_a}	$C_{D_{vol}}$
0.0667	0.14	0.032
.10	.17	.051
.15	.29	.11

where C_{D_a} is the drag coefficient based on the frontal area and $C_{D_{vol}}$ is the drag coefficient based on the volume of the body to the 2/3 power.

The drag of a given volume is an important criterion in the case of an airplane fuselage and it will be of interest to compare these values with a typical value attainable at subsonic speeds. For a Reynolds number of 10^8 and a turbulent boundary layer, the best value given by Young (reference 5) corresponds to a thickness ratio of 0.2 and is approximately

$$C_{D_{vol}} = 0.016$$

CONCLUDING REMARKS

The theoretical pressure distribution over a closed body of revolution traveling at supersonic velocities shows a pressure recovery at the rear of the body similar to that occurring at subsonic speeds. The extent of the region of positive pressure is, however, not sufficient to have a pronounced effect on the wave drag. It appears to be necessary to use extremely slender shapes to obtain total drag values comparable to those of a conventional airplane fuselage at subsonic speeds. For fineness ratios between 10 and 15 the theoretical wave drag is of the same order as probable values of the frictional drag.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., July 8, 1946

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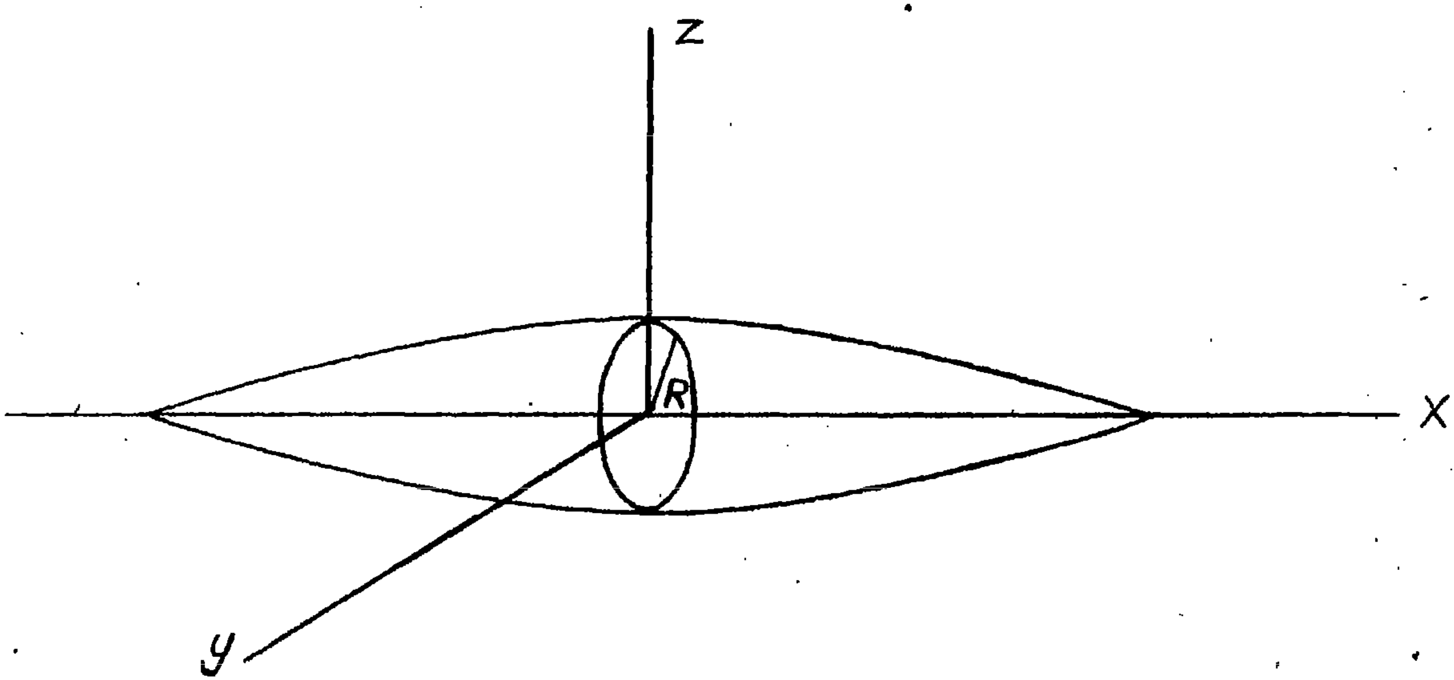

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Figure 1.- Orientation of axes and shape of body.

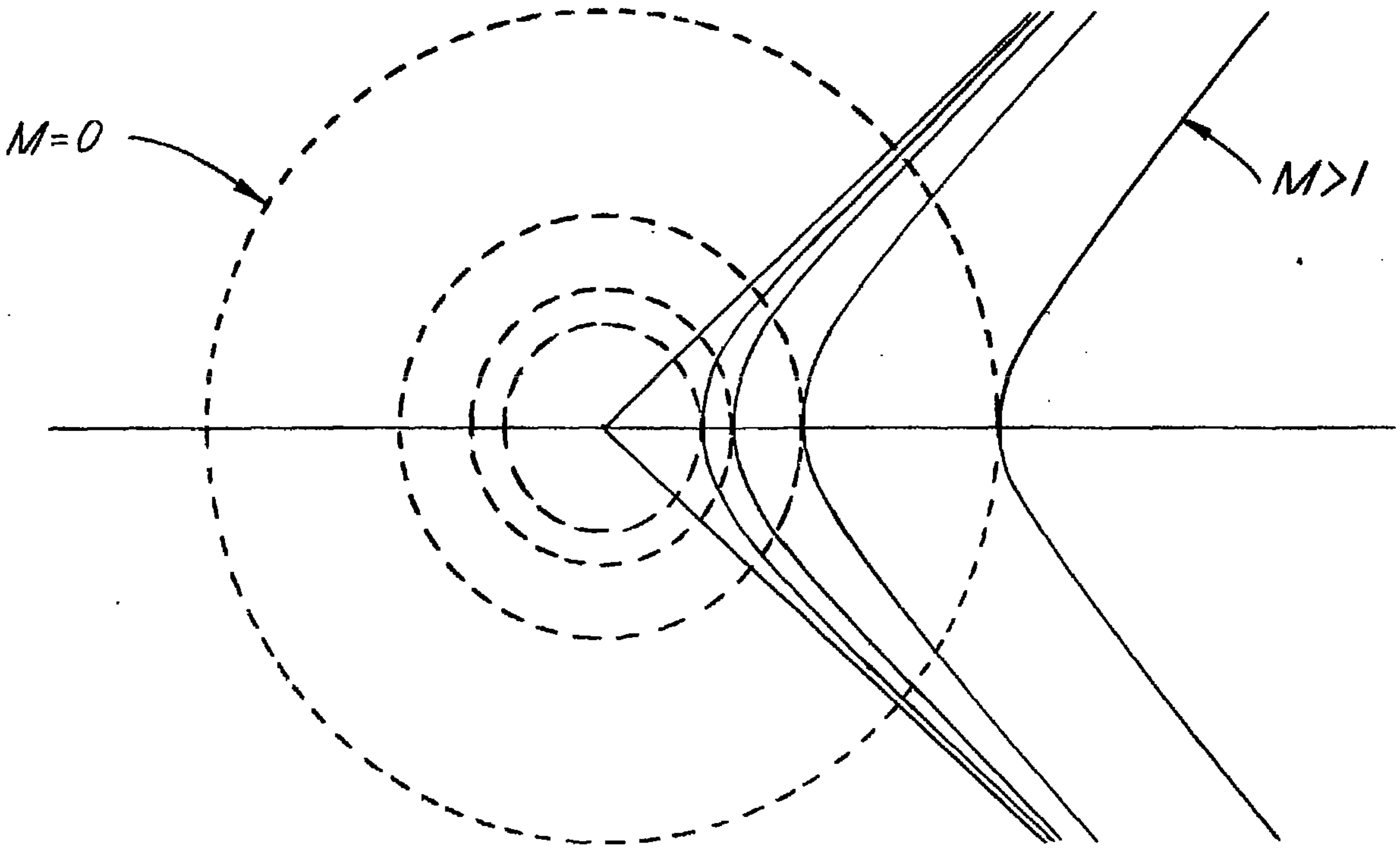

Figure 2.- Equipotential lines for a supersonic source compared with those for a source in an incompressible flow.

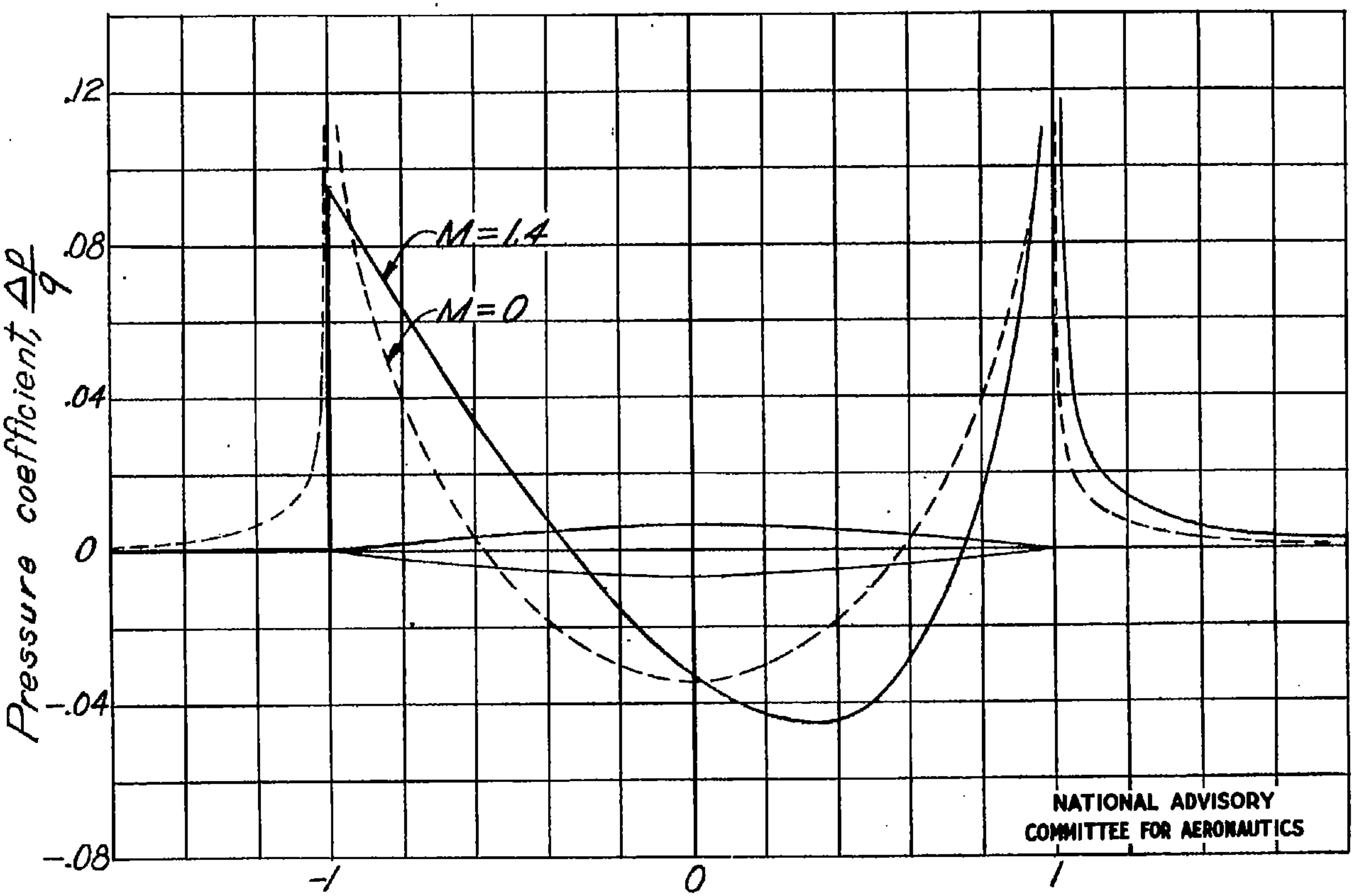

Figure 3.- Calculated pressure distributions for a thickness ratio of 0.0667.

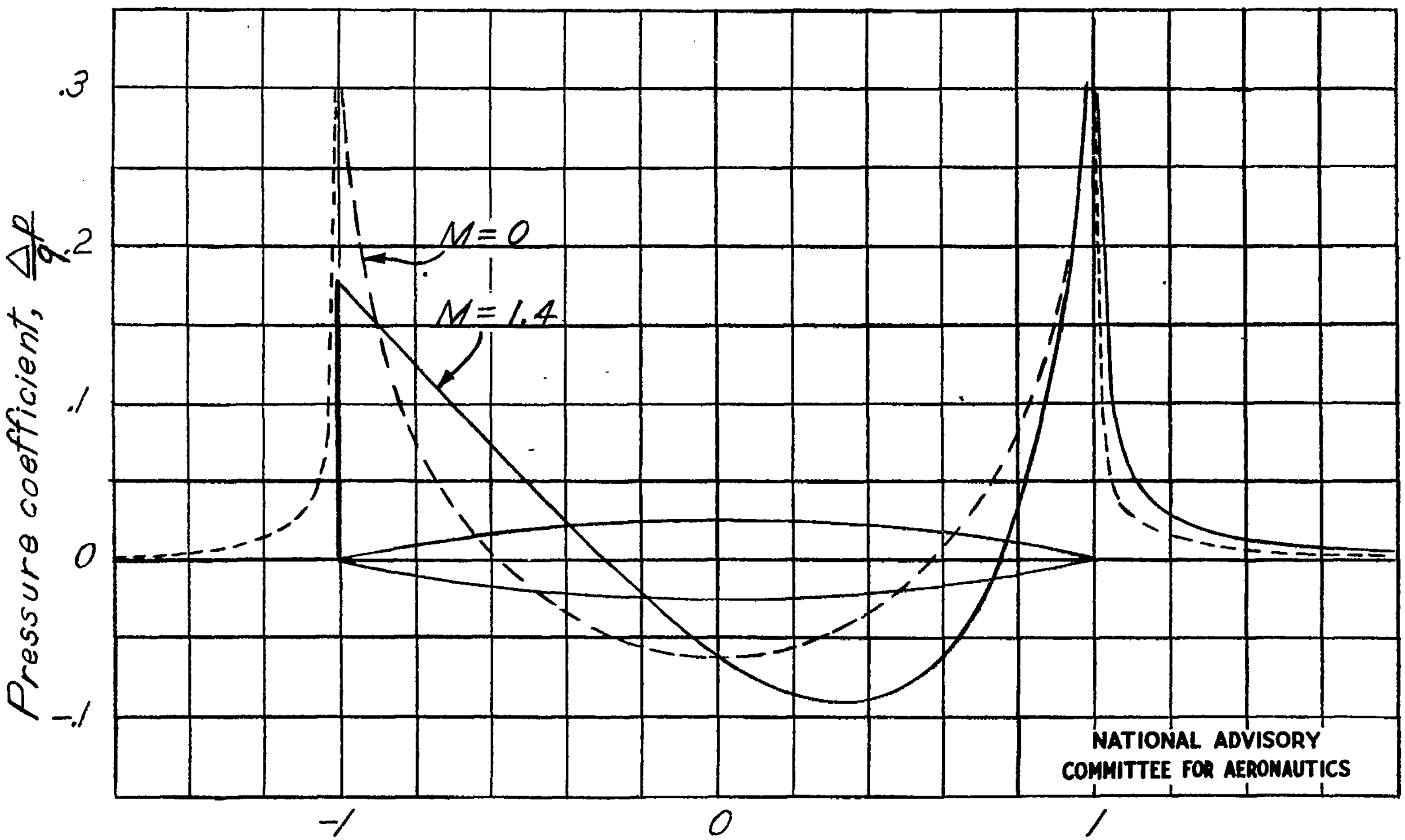

Figure 4.- Calculated pressure distributions for a thickness ratio of 0.10.

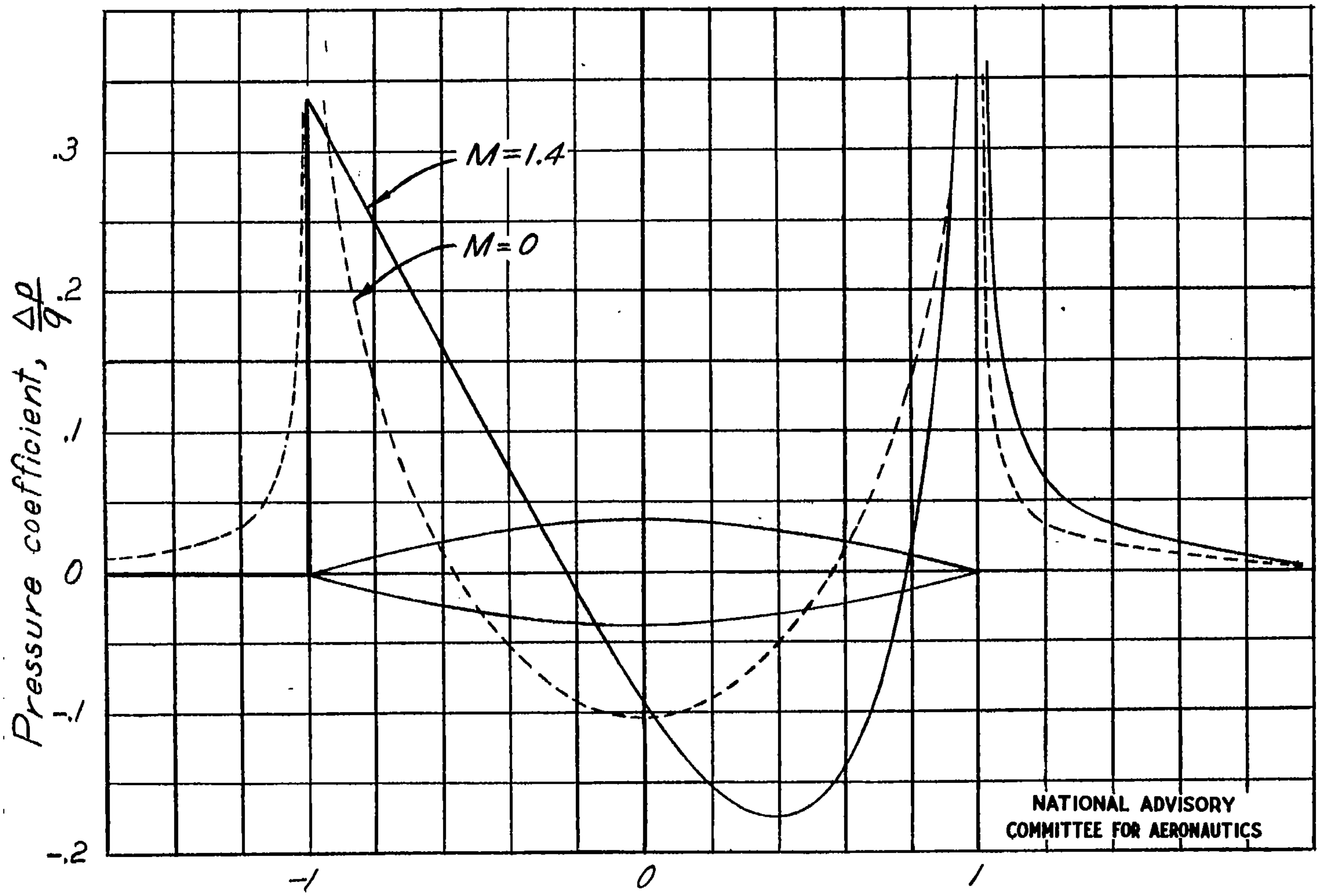

Figure 5.- Calculated pressure distributions for a thickness ratio of 0.15.

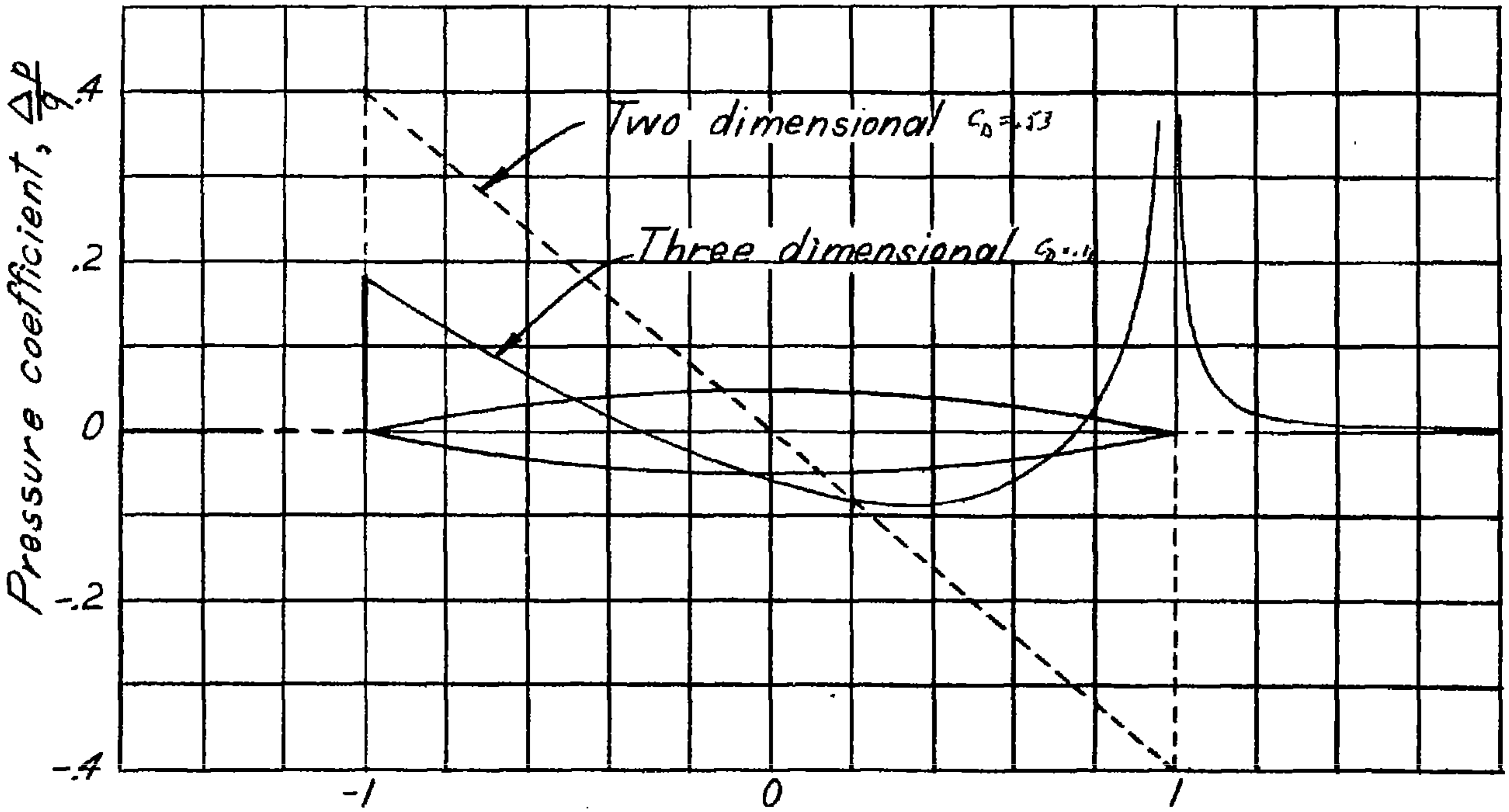

Figure 6.- Comparison of pressures for two- and three-dimensional bodies at a Mach number of 1.4 for a thickness ratio of 0.10.

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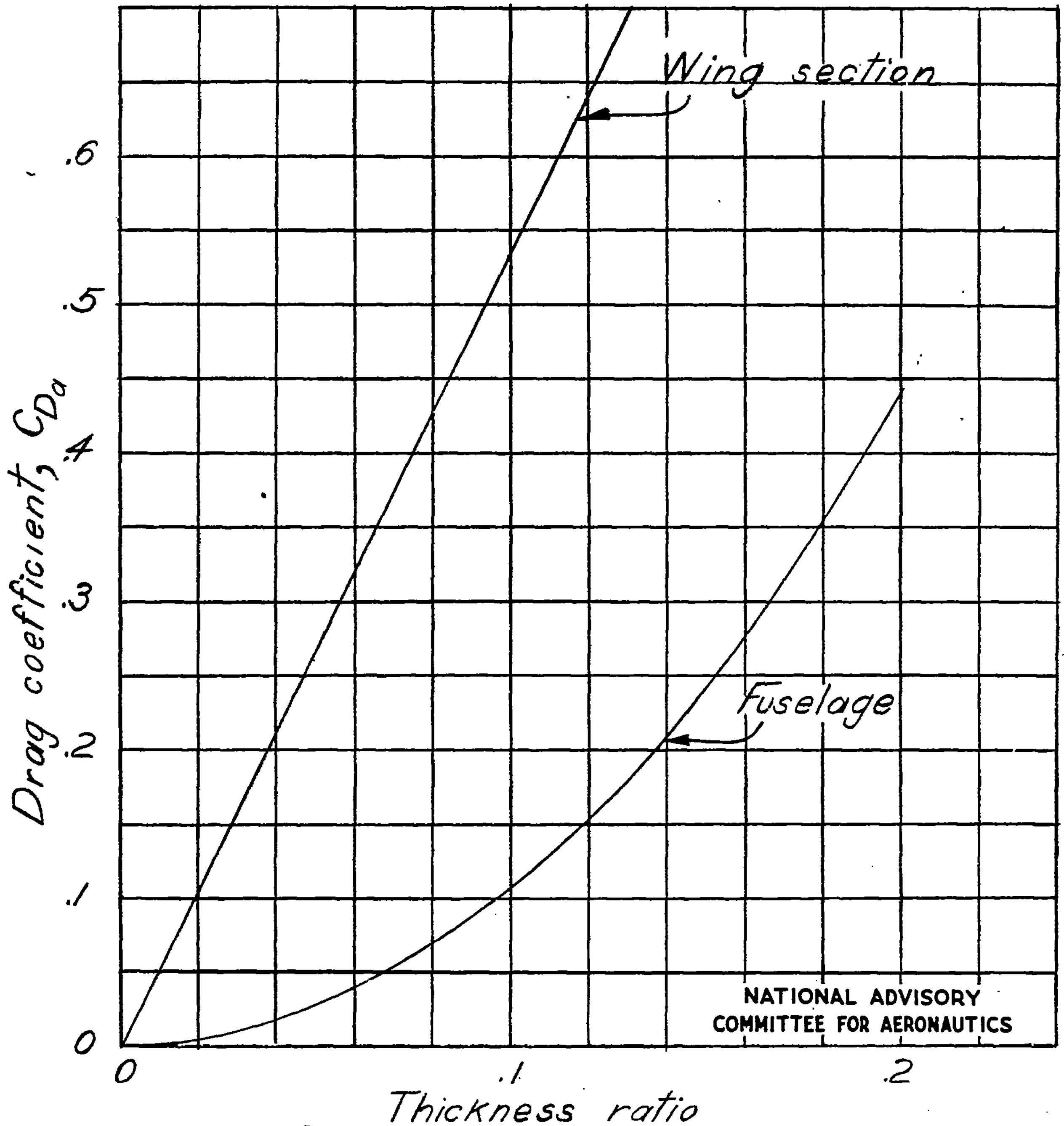

Figure 7.- Comparison of wave drags of two- and three-dimensional bodies for a Mach number of 1.4.